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Date: November 5, 2018

To: EPA AP-42 Review Committee  
US EPA/OAQPS

Subject: AP-42 Chapter 7, Section 7.1 proposed revisions: Organic Liquid Storage Tanks  
Fixed Roof Horizontal Storage Tank Breathing Loss Calculation

### Introduction

These comments offer a more accurate approach for calculating the breathing loss emissions for a horizontal storage tank. These new procedures take the average liquid level of the contents in the vessel into consideration through simple geometry. The existing approach offered in Chapter 7.1 takes into account only the tank dimensions and ignores the actual liquid level height in calculating  $V_v$  and  $H_{v0}$  which can result in significant calculation errors.

### Summary

$V_v$  and  $H_{v0}$  are key variables used in calculating the breathing losses for a fixed roof horizontal storage tank. The existing procedures that are contained Chapter 7.1 are first reviewed. Then, newer equations are proposed which illustrate how  $V_v$  and  $H_{v0}$  can be readily calculated using conventional geometry. Finally, Example 2 from Chapter 7.1 is recalculated using the new approach and the results of the existing and proposed approaches are compared.

### Results

Chapter 7.1 contains Example #2 to illustrate the emission calculations for a fixed roof horizontal storage tank. Example #2 uses similar conditions as were used in Example #1 where both vessels have a shell diameter of 6 feet and a straight side length of 12 feet. The average liquid volume storage in both example problems is the same 1,693 gallons.

When  $V_v$  is calculated using the standard approach (Example #2 in Chapter 7.1), the results are 170 ft<sup>3</sup> and when the equations proposed in this document are applied, the results are 113.6 ft<sup>3</sup>. Also, when  $H_{v0}$  is calculated using the standard approach (Example #2, Chapter 7.1), the results are 2.36 ft and when the equations proposed in this document are applied, the results are 1.63 ft. The calculated results for  $V_v$ ,  $H_{v0}$ ,  $K_s$ , and  $L_s$  using both the standard approach (Example #2, Chapter 7.1) and using the equations proposed in this document are shown in Table 1.

Table I: Comparison between Chapter 7.1 Example 2 and Proposed Procedures

Horizontal Style Vessel	$V_v$	$H_{v0}$	$K_s$	$L_s$
Example #2, Chapter 7-1	170.0 ft <sup>3</sup>	2.36 ft	0.899	57.0 lb/yr
Proposed Procedure	113.08 ft <sup>3</sup>	2.205 ft	0.928	38.07 lb/yr

It is also noted that  $V_v$  was calculated for the vertical cone top tank in Example #1 (Chapter 7.1) to be 114.86 ft<sup>3</sup> and the yearly breathing losses were 36 lb/yr.

## Conclusion

$V_v$  represents the volume of the vapor space that exists in the horizontal storage tank and  $H_{v0}$  represents the average height of the vapor space. The proposed equations enable  $V_v$  and  $H_{v0}$  to be accurately calculated since liquid height is taken into account through the use of geometry. A low liquid level in the tank would result in a high vapor space volume and a high liquid level would result in a low vapor space volume.

The equations provided in Chapter 7.1 do not take the actual liquid height into account. Instead, the vapor space volume ( $V_v$ ) and average vapor height ( $H_{v0}$ ) are calculated based solely on the dimensions of the horizontal storage tank. For the same vessel, this approach will always result in the same value for  $V_v$  and  $H_{v0}$  regardless of whether the horizontal storage tank has a high liquid level or a low liquid level for the same tank.

The yearly breathing losses calculated in Example #2 (Chapter 7.1) are 57 lb/yr. The breathing losses ( $L_S$ ) calculated using the procedures described in this document were 39 lb/yr which is 32% lower than the Example #2 (Chapter 7.1) results. Since the average liquid level in the horizontal storage tank is routinely measured and recorded, then the chemical operator would have all of the information needed to use the proposed calculations.

## Discussion

### Existing Chapter 7.1 procedures for calculating $V_v$ and $H_{v0}$ for a horizontal storage tank

Calculating the standing or breathing losses from a fixed roof storage tank is accomplished using EPA equation Eq (1-2)<sup>1</sup> which takes into account the vapor space volume  $V_v$ , stock vapor density  $W_v$ , vapor space expansion factor  $K_E$ , and vapor saturation factor  $K_S$  as shown in the following expression.

$$L_S = 365 V_v W_v K_E K_S \quad (1-2)$$

Using the existing Chapter 7-1 approach, components  $W_v$  and  $K_E$  are calculated using equations that are independent of whether the tank is a vertical or a horizontal vessel. However, the vapor space volume  $V_v$  and vapor saturation factor  $K_S$  are calculated differently for a vertical storage tank than they are for a horizontal storage tank.  $V_v$  for the vertical tank takes into account vapor space in the vertical cylinder that is above the liquid and the vapor space that is contained by the cone or dome roof. On the other hand,  $V_v$  for the horizontal vessel is calculated using an approximate shell diameter  $D_E$  and approximate vertical height  $H_E$ . The liquid surface area of the horizontal storage tank is calculated assuming that the vessel is at 50% of capacity, where  $A_H = (L)(D)$ . An imaginary vertical cylindrical tank with a flat top is considered to have the same liquid surface area  $A_S$ . An effective shell diameter  $D_E$  of the imaginary vertical cylindrical tank is then calculated using Equation (1-14)<sup>2</sup>:

$$D_E = \sqrt{\frac{A_H}{\pi/4}} = \sqrt{\frac{LD}{\pi/4}} \quad (1-14)$$

The effective height of the imaginary vessel,  $H_E$ , is calculated as the height of an equivalent upright cylinder using Equation (1-15)<sup>3</sup> as shown:

$$H_E = \frac{\pi}{4} D_E \quad (1-15)$$

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<sup>1</sup> AP42 Chapter 7.1, Eq (1-2), page 7.1-15

<sup>2</sup> AP42 Chapter 7.1, Eq (1-14), page 7.1-19

<sup>3</sup> AP42 Chapter 7.1, Eq (1-15), page 7.1-20

$H_{VO}$  is then approximated to be equal to  $\frac{1}{2}$  of  $H_E$ :

$$H_{VO} = \frac{1}{2} H_E$$

Finally, the vapor space volume  $V_V$  is calculated by multiplying the approximate liquid surface area times the approximate vapor height as shown in Eq (1-3)<sup>4</sup>:

$$V_V = \left(\frac{\pi}{4} D^2\right) H_{VO} = \left(\frac{\pi}{4} D_E^2\right) \left(\frac{H_E}{2}\right) \quad (1-3)$$

#### Recommended procedures for calculating $V_V$ and $H_{VO}$ for a horizontal storage tank

The methodology proposed in this section provides a much more accurate approach for calculating  $V_V$  and  $H_{VO}$  for a horizontal fixed roof tank than the approach described in the proposed rule. Figure 1 is a diagram of the circular end of a horizontal storage tank with diameter  $D$  and liquid height  $H_L$ . The length of the horizontal storage tank  $L$  is not shown.

Since  $H_L$  is routinely measured by the chemical operator for a horizontal storage tank as it is for a vertical storage tank, then using the known liquid height  $H_L$  and the dimensions of the vessel ( $L$  and  $D$ ), it is possible to accurately calculate  $V_V$  using conventional geometry.

Radius  $R$  is calculated from the shell diameter  $D$  as shown Eq-1. The total volume of the horizontal storage tank may be calculated by multiplying the length ( $L$ ) of the vessel shell times the area ( $A_T$ ) of the circular end of the shell as shown in Eq-2.

Equations for calculating  $V_V$  of a horizontal storage tank are as follows:

Radius of vessel shell: 
$$R = \frac{D}{2} \quad \text{Eq-1}$$

Calculation of Volume  $V_T$ : 
$$V_T = L A_T = L \pi R^2 \quad \text{Eq-2}$$

The volume of the liquid contents of the horizontal vessel can be calculated using Eq-3 as a function of  $L$ ,  $R$ , and the liquid height  $H_L$ . An explanation for Eq-3 is provided at the end of this document.

Calculation of Liquid Volume  $V_L$ : 
$$V_L = L \left[ R^2 \cos^{-1} \left( \frac{R-H_L}{R} \right) - (R-H_L) \sqrt{2RH_L - H_L^2} \right] \quad \text{Eq-3}$$

The volume of the vapor space can be calculated by subtracting the liquid volume from the total tank volume as shown in Eq-4.

Calculation of Vapor Space Volume  $V_V$ : 
$$V_V = V_T - V_L \quad \text{Eq-4}$$

The vapor space height may be calculated in Eq-5 by subtracting the liquid height  $H_L$  from the shell diameter  $D$  of the horizontal storage tank.

Vapor space height: 
$$H_{VO} = D - H_L \quad \text{Eq-5}$$

#### Example calculation

Example #2 in Chapter 7-1 features the calculation of breathing and working loss for a horizontal storage tank. The tank has a length of 12 ft and a shell diameter of 6 ft. For this example, the same storage conditions are used as in Example 1 of the document. The average daily contents volume used in

<sup>4</sup> AP42 Chapter 7.1, Eq (1-3), page 7.1-15

Example #1 was 8 ft which calculates to be 226.19 ft<sup>3</sup> (1,693.2 gal). If the horizontal vessel in this example is filled with 226.19 ft<sup>3</sup> or 1,693.2 gal of liquid, then the liquid height H<sub>L</sub> would be 3.795 ft. Since H<sub>L</sub> is normally measured by the chemical operator, this value will be known.

Given: Shell diameter D: 6 ft  
 Shell Length L: 12 ft  
 Liquid height H<sub>L</sub>: 3.795 ft

Radius of vessel shell:  $R = \frac{6 \text{ ft}}{2} = 3 \text{ ft}$  Eq-1

Liquid Volume V<sub>L</sub>:  $V_L = L \left[ R^2 \cos^{-1} \left( \frac{R-H_L}{R} \right) - (R-H_L) \sqrt{2RH_L - H_L^2} \right]$  Eq-3

$$V_L = 12 \left[ 9 * 1.839 - (-0.796) \sqrt{22.77 - 14.40} \right]$$

$$V_L = 12 * 18.851 = 226.21 \text{ ft}^3$$

Vapor Space Volume V<sub>V</sub>:  $V_V = V_T - V_L$  Eq-4

$$V_V = 339.29 \text{ ft}^3 - 226.21 \text{ ft}^3 = 113.08 \text{ ft}^3$$

Distance from liquid surface to top of tank H<sub>VO</sub>:  $H_{VO} = 6 \text{ ft} - 3.795 \text{ ft} = 2.205 \text{ ft}$  Eq-5

$$K_S = \frac{1}{1+(0.053)(P_{VA})(H_{VO})}$$

$$K_S = \frac{1}{1+(0.053)(0.901)(2.205)} = 0.905$$

Using the newly calculated values for V<sub>V</sub> and K<sub>S</sub> from the proposed equations in conjunction with the established values of W<sub>V</sub> and K<sub>E</sub>, the breathing losses for the horizontal storage tank can be calculated.

Vapor density (Chapter 7, Step 4b, example 2)<sup>5</sup>:  $W_V = 1.29 \times 10^{-2} \text{ lb/ft}^3$

Expansion coefficient (Chapter 7, Step 4c, example 1)<sup>6</sup>:  $K_E = 0.079$

Calculation of breathing loss<sup>7</sup>:  $L_S = 365 V_V W_V K_E K_S$

$$L_S = 365 (113.08)(1.29 \times 10^{-2}) (0.079) (0.905)$$

$$L_S = 38.07 \text{ lb/yr}$$

Using the proposed approach outlined in this document, the yearly breathing losses for the horizontal storage tank are calculated to be 38.07 lb/yr. The yearly breathing losses that were calculated in AP-42 Chapter 7-1 for Example 2 were 57 lb/yr. The results from the Chapter 7-1 procedures are significantly greater than when actual liquid contents measurements and conventional geometry are used. This over estimation of breathing losses is primarily because the vapor space volume V<sub>V</sub> (170 ft<sup>3</sup>) (Chapter 7-1) is calculated using approximate vessel dimensions and V<sub>V</sub> (113.08 ft<sup>3</sup>) which was calculated using the equations proposed in this document accurately reflect the true vapor space volume of the horizontal tank.

Development of the equation for calculating the liquid volume in a horizontal storage tank

<sup>5</sup> AP-42 Chapter 7-1 (revisions), page 7.1-159

<sup>6</sup> AP-42 Chapter 7-1 (revisions), page 7.1-159

<sup>7</sup> AP42 Chapter 7.1, Eq (1-2), page 7.1-15

The methodology proposed in this document provides a much more accurate approach for calculating  $V_v$  and  $H_{v0}$  for a horizontal fixed roof tank than the approach described in the proposed rule. Figure 1 is a diagram of the circular end of a horizontal storage tank with diameter  $D$  and liquid height  $H_L$ . The length  $L$  of the horizontal storage tank is not shown.

Using the known liquid height  $H_L$  and the dimensions of the vessel ( $L$  and  $D$ ), it is possible to accurately calculate  $V_v$  using standard geometric equations.

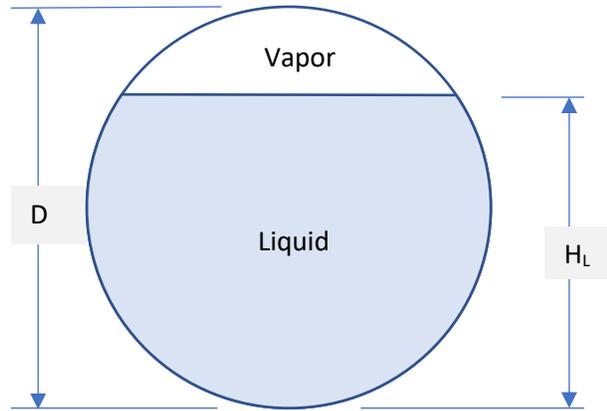


Figure 1: Circular End of a Horizontal Storage Tank

The methodology for calculating the volume of liquid in the horizontal storage tank  $V_L$  takes into account the length of the shell ( $L$ ) and the area ( $A_L$ ) of the circular end of the vessel that is covered by the liquid contents.

Figure 2 shows the circular end of a horizontal storage tank where the liquid height ( $H_L$ ) is approximately 25% of the available height (or diameter) of the shell. The surface of the liquid intercepts the shell at points A and C. A circular sector is created by two line segments with length  $R$  that extend from the center at point A to intercept the shell at points B and C. The area of the circular sector  $A_s$  is proportional to the ratio between  $\theta$  and the total number of radians in the circle ( $2\pi$ ). Therefore, if  $\theta$  is known, then the area of the circular sector  $A_s$  calculated using Eq-6.

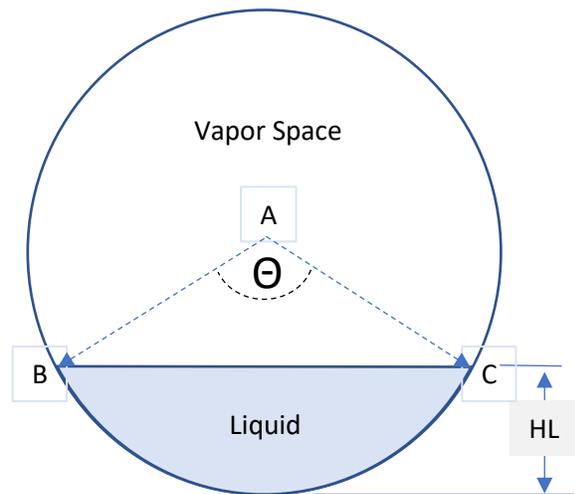


Figure 2: Circular End of a Horizontal Storage Tank

Area of the circular sector: 
$$A_s = \frac{\theta}{2\pi} A_c \tag{Eq-6}$$

where  $A_s$  is the area created by the circular sector ABC,  $\theta$  is the angle created by AB and AC in radians.

Figure 3 shows the circular end of a horizontal storage tank where the liquid height ( $H_L$ ) is approximately 75% of the available vessel height of the shell. In this example, the circular sector is the shaded portion of the circle that is below the liquid surface and excludes the area  $A_T$  of the triangular section created by points A, B, and C. Assuming that  $\theta$  is known, then the area  $A_S$  in Figure 3 may be calculated using Eq-6 as described earlier.

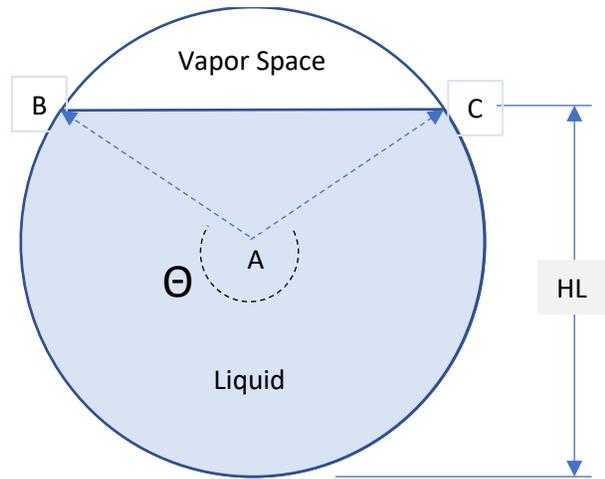


Figure 3: Circular End of a Horizontal Storage Tank

#### Calculation of $A_S$ the area of the circular sector:

Area of the circular sector  $A_S$  shown in Figure 2 and Figure 3 may be calculated by multiplying the area of the circular shell by the ratio between the circular sector angle  $\theta$  (in radians) and the total number of radians in the circular shell as shown in Eq-6.

$$\text{Area of the circular sector:} \quad A_S = \frac{\theta}{2\pi} A_C \quad \text{Eq-6}$$

$$\text{Area of circular shell:} \quad A_C = \pi R^2 \quad \text{Eq-7}$$

The triangular section,  $\Delta ABC$ , shown in Figure 2 and Figure 3 may be subdivided into two smaller triangles using a perpendicular line that connects from the center of the liquid surface at point D to the center of the shell circle at point A. Since  $\Delta ABC$  is an isosceles triangle, then the two smaller angles  $\phi$  that are created at point A are equal and one half the size of the original angle  $\theta$  at point A.

$$\theta = 2\phi \quad \text{Eq-8}$$

$$A_S = \frac{\theta}{2\pi} A_C = \frac{2\phi}{2\pi} \pi R^2 = \phi R^2 \quad \text{Eq-9}$$

The length (h) of the vertical line that runs perpendicular to the liquid surface and connects to the center of the circular shell A may be calculated by subtracting the height of the liquid surface  $H_L$  from radius R of the circular shell.

$$\text{Length of vertical line} \quad h = R - H_L \quad \text{Eq-10}$$

$\phi$  may be calculated as the arccosine of the ratio between h and R as shown in Eq-11 and further simplified in Eq-13 to be a function of only R and  $H_L$ .

$$\phi = \cos^{-1}\left(\frac{h}{R}\right) = \cos^{-1}\left(\frac{R-H_L}{R}\right) \quad \text{Eq-11}$$

Finally, the area of the circular sector  $A_S$  is calculated by substituting Eq-11 into Eq-9 as shown in Eq-13.

$$A_S = R^2 \cos^{-1}\left(\frac{R-H_L}{R}\right) \quad \text{Eq-13}$$

### Calculation of the area $A_T$ of triangle $\Delta ABC$ .

The perpendicular distance from the center of the liquid surface to the center of the circle can be calculated by subtracting  $H_L$  from  $R$ .

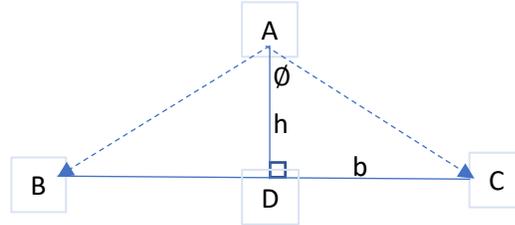


Figure 4: Triangular portion from Figure 1 and Figure 2.

Previously, two smaller triangles  $\Delta ABD$  and  $\Delta ACD$  were created from the larger triangle  $\Delta ABC$  when a vertical line was drawn perpendicular from the center of the liquid surface to the center of the circle at point A as shown in Figure 4.

$A_T$  can be calculated as the sum of the two smaller triangles  $\Delta ABD$  and  $\Delta ACD$  as shown in Eq-14 where  $b$  and  $h$  are the same value for each triangle.

$$\text{Area of } A_T: \quad A_T = 2 \left( \frac{1}{2} b h \right) = b h \quad \text{Eq-14}$$

$$\text{Expression for } h \text{ earlier:} \quad h = R - H_L \quad \text{Eq-10}$$

Side  $b$  of triangles  $\Delta ABD$  and  $\Delta ACD$  may be calculated using the Pythagorean theorem for a right triangle as shown in Eq-15.

$$\text{Calculation of } b^2: \quad b^2 = R^2 - h^2 = R^2 - (R - H_L)^2 \quad \text{Eq-15}$$

$$b^2 = R^2 - (R^2 - 2 R H_L + H_L) = +2RH_L - H_L \quad \text{Eq-16}$$

$$\text{Calculation of } b: \quad b = \sqrt{2 R H_L - H_L} \quad \text{Eq-17}$$

$h$  as defined in Eq-10 and  $b$  as defined in Eq-17 are substituted into Eq-14 to create an equation to calculate the area of  $A_T$  in terms of only  $R$  and  $H_L$  as shown in Eq-18.

$$\text{Calculation of } A_T: \quad A_T = (R - H_L) \sqrt{2RH_L - H_L} \quad \text{Eq-18}$$

Finally, the area  $A_L$  of the portion created by the liquid at the circular end of the shell may be calculated by subtracting the area of the triangular section  $A_T$  from the area of the triangular area  $A_T$  in Eq-19.

$$\text{Calculation of } A_L: \quad A_L = A_S - A_T \quad \text{Eq-19}$$

Eq-19 can be further revised so that the area created by the liquid at the circular end may be calculated using only the values for  $R$  and  $H_L$ .

$$\text{Calculation of } A_L: \quad A_L = R^2 \cos^{-1} \left( \frac{R-H_L}{R} \right) - (R - H_L) \sqrt{2RH_L - H_L} \quad \text{Eq-20}$$

Note that if the liquid height is below the center of circular shell where  $H_L$  is lower than  $R$  (as seen in Figure 2), then the value for  $A_T$  is subtracted from the area of the circular sector. Alternatively, when the liquid height is above the center of the circular shell where  $H_L$  is greater than  $R$  (as seen in Figure 3), then the value for  $A_T$  is added to  $A_S$ .

The volume of the liquid contained in the horizontal storage tank may be calculated by multiplying the length of the shell by the area created by the liquid at the circular end of the shell as shown in Eq-21.

Calculation of  $V_L$ : 
$$V_L = L \left[ R^2 \cos^{-1} \left( \frac{R-H_L}{R} \right) - (R - H_L) \sqrt{2 R H_L - H_L^2} \right] \quad \text{Eq-20}$$

The vapor space volume may now be calculated by subtracting the liquid volume from the total volume of the horizontal storage tank as shown in Eq-21.

Calculation of  $V_V$ : 
$$V_V = V_T - V_L \quad \text{Eq-21}$$

### Definition of terms

- D Diameter of circular end of the horizontal storage tank shell.
- R Radius of circular end of horizontal storage tank shell.
- $V_V$  Vapor space volume in horizontal storage tank.
- $V_T$  Total capacity volume of horizontal storage if 100% full.
- $V_L$  Volume of liquid in storage tank.
- $H_{VO}$  Vapor space height in storage tank.
- $H_L$  Height of liquid in storage tank.
- $A_S$  Area of circular sector created by two line segments that pass through the center of the circular end of the tank and intercept the liquid surface at either side.
- $A_C$  Total area of circular end of the storage tank.
- $\Theta$  Angle in radians that is created by two line segments that pass through the center of the circular end of the tank and intercept the liquid surface at either side.